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The development of mathematical understanding

The Carawatha Language Development Centre

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The Carawatha Language Development Centre attempted to provide the practical links of the Gawned model (see article this issue), by implementing it into the classroom context. The teachers believed the transition from 'real world' mathematics to literate mathematics and its specific abstract language and concepts was not automatically achieved by many children. The teachers sought to use scaffolding to enable children to make a smooth transition between the two and to see the connections between the two.

The Carawatha Language Development Centre is a gazetted Special School which provides specialised language intervention on an intensive basis for children with normal non-verbal cognitive functioning, whose academic, emotional and social performance is severely limited by a profound neurological language delay/disorder.

As a major area of school development the staff of the Carawatha Language Development Centre decided to research current studies on the importance of the language of maths in the learning process and apply this in conjunction with our own existing knowledge of language to the maths curriculum.

Language development

Language development occurs along a continuum from the communicative, oral language of the home founded in the domain of concrete

operations to an increasingly more decontextualised literate, written language. According to Westby (1985), this development involves:

- function – why people talk
- topic – what people talk about; and
- structure – how people talk.

Many children, be they language delayed or mainstream, experience great difficulty in coping with the language of the classroom, particularly as it becomes more abstract (Blank 1978). Experiences must be provided to encourage a natural transition from oral language to functional literate language if the child is to achieve success at school. The authors believe that the Carawatha model provides a continuum on which children across the grades can be placed(see p. 65).

Mathematical development

When children are learning to talk they are confronted with language as a whole and this evolves along a continuum of developing complexity. It would seem logical to assume that children should also be allowed to gain a concept of mathematics as a whole and that mathematical learning should develop in a similar way.

The beginning of mathematical understanding commences at birth, and develops as children experience their world as a place of order, pattern and prediction. In these pre-school years, usually through the medium of play, the children become involved in activities such as labelling, sorting, classifying, sequencing, comparing, measuring and problem solving. Through these processes they will come to terms with the mathematical concepts of grouping, ordering and transforming.

As a result of this natural process of discovery, the foundations of mathematical learning are formed. Unfortunately this prior knowledge is often segmented into distinct, often unrelated units of mathematical learning when the child begins formal education; and the total concept of mathematics, particularly the language of maths, as being a functional part of everyday life is destroyed.

The Carawatha model

In an endeavour to avoid this splintered approach, and to ensure that there is a cohesive and functional policy to match the curriculum expectations across all grades, the staff developed a model based on a continuum of mathematical language development.

Objectives of the Carawatha model

- To establish a continuation of the development of understanding that occurs through all stages of learning.
- To define the importance of recognising the teaching of the language of mathematics in the learning process.
- To assist teachers entering the Centre to recognise the significance of the language of mathematics.
- To form a continual reminder that mathematical concepts and language should develop from exploration and experience at all grade levels and then progress to symbolic representation.
- To ensure that children commence new mathematical learning at a position along the continuum that matches their knowledge and ability rather than at a position designated by chronological age and curriculum expectations.
- To provide a framework for evaluation that:
  - ~ is appropriate to all grades
  - ~ avoids over emphasis on being right or wrong
  - ~ encourages discovery and initiative
  - ~ provides positive feedback
  - ~ allows the transfer of information and the sharing of ideas between all staff members.

The application of this model should ensure that the language of mathematics and the learning/teaching strategies used at the classroom level are inter-related and at the same time determine whether, as a school, staff are addressing the language of mathematics in day to day teaching.

Development of the Carawatha model

Although in its early stages of application, a number of important points are emerging from the model as staff focus on the four critical elements that provide meaning in maths, i.e. Problem Solving Language, Activity Specific Language, Language of the Maths Curriculum and Literacy in Mathematics (Gawned 1990, this issue).

- Maths and the language of mathematics is an integral part of all teaching and learning.
- There is not an automatic transition from real world maths to literate maths with its specific abstract language and concepts.
- Teachers need to provide the scaffolding to enable the child to make a smooth transition and see the connections between the two.
- Concrete experience and exploration is essential in the upper grades

THE CARAWATHA MODEL

The Language of Mathematics

THE LANGUAGE OF MATHEMATICS

Real World	Everyday language of Mathematics	Language of Maths Curriculum	Literacy in Mathematics
Activity Specific Language ←→	**Problem Solving Language ←→**	**Language of Maths Curriculum ←→**	**Literacy in Mathematics**
• Vocabulary e.g. Labels Location • Attributes (Colour, shape, size) • Function • Description e.g. *Put in* *stack* *take away* *collect* • Relationships e.g. *match* *fit* *same as in/on* • Responding	• Absorbing • Reasoning • Predicting • Exploring • Inferring • Comparing • Negating • Decision Making • Selecting • Justifying • Evaluating/True/False • Summarising • Clarifying • Generalising	Set Subset Intersection Frames Diagrams Equality Language of: numeration measurement space time	Representations Using/Recordin in maths Symbolic Language Drawing Diagram Numerals Formulae
NON SPECIFIC TERMINOLOGY		SPECIFIC TERMINOLOGY	

Note: For convenience sake this model has been placed along a continuum of developing mathematical understanding. It must be understood that these areas are continually interactive and their position on the continuum is not prescriptive but dependent upon intellectual capability, prior experience, knowledge required and the pre-determined outcomes of the process being taught.

in order for students to gain a true understanding and functional use of maths.

- There is a need to establish 'base lines' to avoid assumptions of prior knowledge or a lack of a known concept. Questions must be asked as to what the children already know, and which techniques can be used to determine their existing knowledge and informed decisions made in regard to what children need to learn.
- Evaluation techniques should be designed to measure the conceptual and language knowledge of the mathematical process being attempted. Often it is the child's inability to use the language of the task to explain a concept well understood that results in a teacher's belief that the child has failed to learn.
- Care should be taken that the fine and gross motor skills required for a task do not detract from the concept being taught. Children with difficulties in these areas may need to concentrate solely on the motor co-ordination required for some activities and in doing so fail to meet the specific demands of the lesson.

Application of the model

In order to provide practical links between the model and its classroom application, a series of activities will be presented to demonstrate how the language of mathematics can be developed along the continuum between the real world and maths literacy and create a learning environment that ensures that the teaching of maths is successful, meaningful and enjoyable.

MATH STRAND – NUMBER

Objective: Take a given number of objects from a set.

Focus: Real world → Language of maths curriculum.

Activity

Ingredients

- 9 eggs
- 8 cups of flour
- 7 piles of sultanas
- 2 cups of milk
- 4 cups of sugar

Activity Specific Language

We are going to make some biscuits.
Here are all the things we need.
(Show ingredients, name and discuss.)

Problem Solving Language

(Our recipe doesn't need all of these supplies.)
How many eggs are there? Count. (Answer: 9)

Eggs We need two eggs.
Take away two eggs. How many are left? (Answer: 7)

Flour How many cups of flour are there?
Guess without counting. (Answer: 8)
We need four cups of flour
Take away four cups of flour.
How many are left? (Answer: 4)

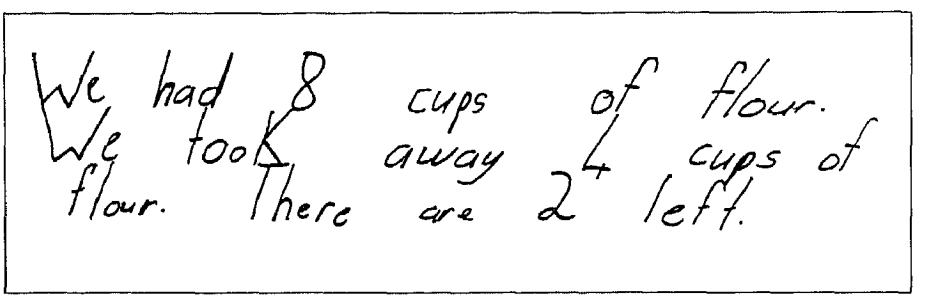
Follow a similar procedure for all ingredients.

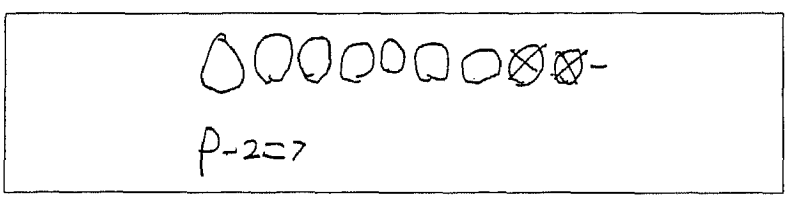
Language of Math Curriculum

Ask the children to record what they did.

Recording

Recording procedures are not prescriptive but allow the children to record their own perceptions in their own way. Symbolic representation is optional.





Samples

Selecting the activity

- When selecting mathematical activities teachers must have a set focus and an established knowledge of the outcomes required.
- Instructions must be simple and specific to the task being set.
- Appropriate questions must be asked at the beginning of the activity to ensure that children understand what is required of them.
- Strategies to remind children of the task set should be incorporated into the activity.
- Children should be allowed to find their own conclusions where possible and not have an adult's pre-conceived outcome imposed upon them.
- Children should be encouraged to record their findings orally, pictorially, in written or symbolic form.

Introducing the activity

Children should be allowed to develop a knowledge of the structure of the game/activity. Initially the teacher will see the mathematical focus but the children will see the game to be played.

It is important that children have the opportunity to handle and examine the materials involved and have time to learn the rules of the game. When this knowledge is established the children will begin to focus on the mathematical concept and connected language for which the activity was initially planned (Pengelly, H. 1990).

Repeating the activity

Activities can be repeated or adapted to meet the demands of a new focus and outcome whilst still retaining the children's existing knowledge of the resources needed and the rules of the game.

Teachers have a tendency to use an activity once and then discard it. If it was successful use it again and again. How many times do we play Snakes and Ladders, Chinese Checkers, Noughts and Crosses, Monopoly® and card games with continued enjoyment and increasing skill?

Practical application

MATH STRAND – MEASUREMENT (1)

Objectives: Early time experience using tockers

Tockers provide experience in time measurement for younger children. They have a circular base and a triangular apex. When the apex of the triangle is placed down on a hard surface and released, the tocker rocks on its circular base. When the tocker stops rocking it has measured a small amount of time, usually 5, 10 or 15 seconds.

Focus: Real world → Literacy in Math

Activity Specific Language

- Superlatives and comparatives
- Discussion techniques
- Organisational language.

Problem Solving Language

1 Classifying duration
2 Organising role for group members
3 Working co-operatively.
4 Developing recording methods
5 Comparing and evaluating methods

Language of Math Curriculum

- Time terminology
- Cardinal number

Literacy in Math

- Developing recording techniques
- Evaluating efficiency of recording methods
- Cardinal numerals.

Activity 1

Children experiment with a selection of tockers, comparing the duration of movement. Tockers are classified into long, medium and short duration. Children organised themselves into small groups, each group taking one tocker of the same duration. Children find out and record how many hops, jumps and steps they can do in the time marked by the tocker.

Recording

Recording materials used may be concrete or graphic. On completion, children join the large group to share their findings and discuss their recording methods. Follow-up talk also considers reasons for differing results between groups.

Recording samples are kept in a file to show development throughout the year.

MATH STRAND – MEASUREMENT (2)

Objective: Early time and distance experience using tockers and trundle wheel.

Focus: Real world → Literacy in Math

Activity Specific Language

- Comparatives and superlatives
- Use of questions: 'How + long, far?'
- Organisational language
- Discussion techniques.

Problem Solving Language

1 Forming small, even groups
2 Devising a running order
3 Stopping and starting on signal
4 Measuring using a trundle
5 Working co-operatively
6 Developing recording methods.

Language of Math Curriculum

- Time terminology
- Length/distance terminology
- Cardinal and ordinal number.

Literacy in Math

- Use of recording methods
- Evaluation of efficiency recording methods
- Cardinal and ordinal numerals.

Activity 2

Short classroom activity uses a 5 second tocker to test how many actions can be done in the given time. Main activity takes place outside, preferably on the school oval. Children form themselves into small groups and devise a running order. Using the 5 second tocker, the teacher signals for each child to start and stop running in the given duration. Each child sits on the track at the place they stopped running. The distance that each child ran is then measured using a trundle wheel. This is organised so that each member has a turn at measuring distance for another class member.

Recording

The initial recording of distance is done on paper by each individual. On returning to the classroom, the small groups reform and work on recording the distance that all the group members ran. Graphic and concrete materials are available for use. Results and recording methods

are shared with the whole group. Recording samples are kept in a file to show development throughout the year.

MATH STRAND – MEASUREMENT (3)

Objectives: Early time and length experience using stopwatch and trundle wheel.

Focus: Real World → Literacy in Math

Activity Specific Language

- Comparatives and superlatives
- Use of questions: 'How + long, far, quick fast...?'
- Organisational language
- Discussion techniques.

Problem Solving Language

Problem Solving:

1 Measuring distance using trundle wheel
2 Forming small, equal groups
3 Organising order of children running
4 Initial recording of time
5 Working co-operatively
6 Developing group recording methods.

Language of Math Curriculum

- Time terminology
- Length terminology and ordinal number

Literacy in Math

- Use of recording methods
- Evaluation of efficiency of recording methods
- Cardinal and ordinal numerals.

Activity 3

Short classroom activity uses a stopwatch to time how long simple actions take to complete. Main activity takes place in the outside area, preferably on the sports field. Children use a trundle wheel to measure twenty metres on the running track. Children form themselves into small groups. Each member of each group runs the marked distance and is timed by the teacher using the stopwatch.

Recording

As each child finishes they sit with their small group to make an initial record of their running times on paper. Children return to the classroom to work in their small groups. They devise recording methods that show all the times of the group members. Graphic and concrete materials are available for use. Follow-up discussion shares the recording methods of the small groups with the whole class.

MATH STRAND – NUMBER (1)

Objective: To develop math narrative 'The Take-Away Thief'

Focus: Real World → Language Math Curriculum

Activity 1

Resources

- 15–18 Cup Cakes cooked by children at a prior lesson.
- Mask and Hat for Take-Away Thief.

Activity Specific Language

Tell the children a story about the Take-Away Thief who steals cakes from the Baker's shop when no one is looking.

Set the scene for the Baker's Shop and put out the plate of cakes. Choose one child to be the Take-Away Thief. He/she creeps in and steals 2 or 3 cakes and runs off with them. The class then discuss what has happened.

Another child is selected as the Take-Away Thief and the same procedure is followed until all the cakes have disappeared.

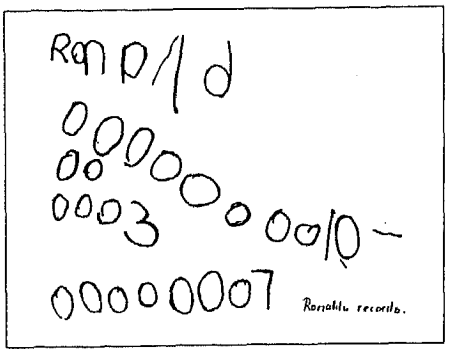
Problem Solving Language

- How many cakes does the baker have?
  Estimate, then count to verify. (Answer 15)
- Predict: ~ How many cakes will be stolen?
  ~ Will there be more cakes or fewer cakes at the end of this game?

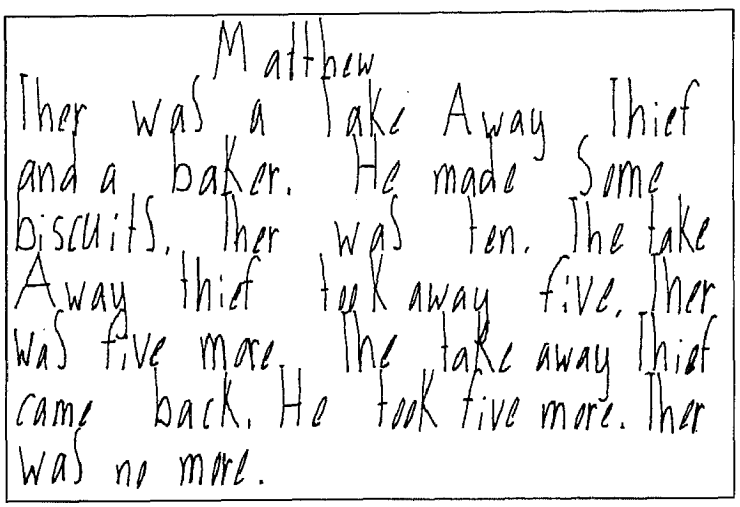
Language of Math Curriculum Recording

Again children should be asked to record what they have done but the recording process should not be determined.

Sample 1



Sample 2



Sample 3

MATH STRAND – NUMBER (2)

Objective: To develop a story from a set algorithm.

Focus: Literacy in Mathematics ← to the Real World.

Activity 2

Literacy in Mathematics

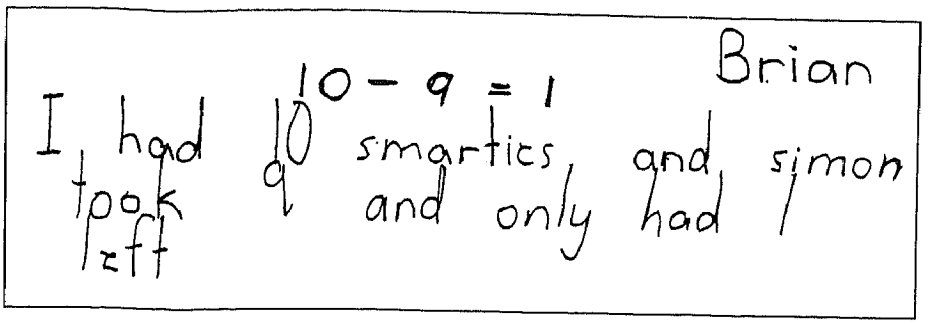
Set number sentence 16 – 7 = 9.

Problem Solving Language

- Group Discussion
- What sort of story could we make up about this sum?

Real World

Sample



MATH STRAND – SPACE

Objective: To investigate boundaries.

Focus: Real World → Literacy of Math

Activity Specific Language

- Vocabulary associated with boundaries and pathways.
- Labels attached to road signs.
- Attributes which describe natural surroundings.
- Framing questions

Problem Solving Language:

- Estimation of time and distance.
- Predictions associated with movement along roadways, inside other boundaries and surrounding bushland.
- Decision making related to position and direction.

Language of Math Curriculum

- Terminology of position, time and length.

Literacy in Math

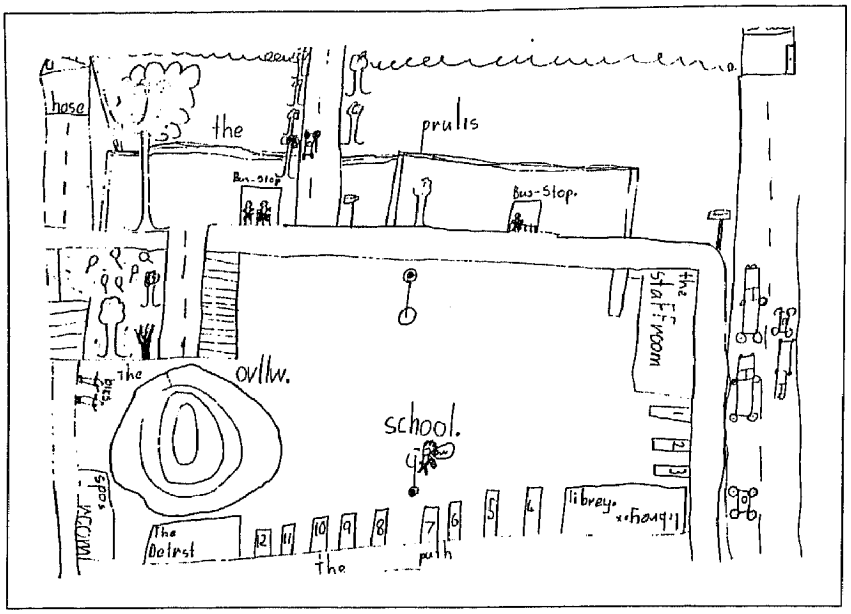
- Recording methods.

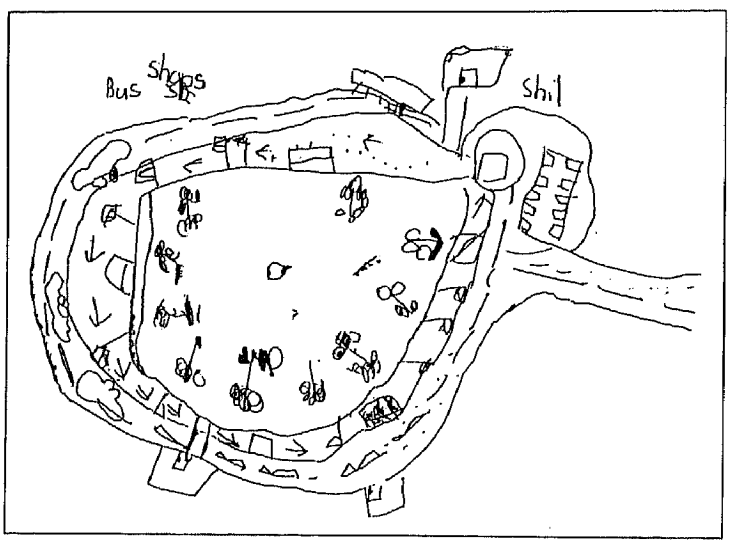
Activity

As part of an integrated them on 'Myself and my community', the whole group leaves school grounds via pathway. Using consensus, a decision concerning choice of pathway is made. Guesses and clarification are made about road signs, street names and house numbers. Estimation in footsteps from various points of reference are also undertaken.

Recording

1 Initial recording was oral.
2 Pictorial recordings of activity were filed.
3 Recordings using concrete play material were encouraged.
4 Further pictorial records were kept and compared to others.
5 Symbolic recordings were noted throughout the developmental process.





Sample recordings: Maths strand 'Space'

Conclusion

As these activities illustrate, mathematics should not be seen as computation or the need to provide the right answer. Literacy without understanding is not functional maths and the links between the two are embedded in language and concrete experience.

The mathematics curriculum should be seen as a valuable resource for desired outcomes across the grades but effective learning will depend upon how the curriculum content is interpreted, planned and applied along a logical continuum of increasing complexity.

The application of this model has provided information about our classroom practice and equally, classroom practice has helped us to explore, elaborate and refine the model to ensure that the children are able to communicate mathematically.

References

Blank, M., Rose, S., Berlin, L., (1978) *The Language of Learning*, Grune and Stratton, New York.

Gawned, S., (1989) 'The Relationship of Language Learning to Mathematics Learning', unpublished article.

Pengelly, H. (1990) Paper presented at Mt. Pleasant P.S. (WA) 1990.

Westby, C. (1985) 'Learning to Talk – Talking to Learn: Oral-Literate Language Differences' in Simon, C. (Ed.), *Communication Skills and Classroom Success*, College Hill Press.